Minimal complexity of MaxFlow on temporal graphs

Yoann Pigné, LITIS, Le Havre. yoann.pigne@univ-lehavre.fr
Eric Sanlaville, LITIS, Le Havre. eric.sanlaville@univ-lehavre.fr
Mathilde Vernet, LIA, Avignon. mathilde.vernet@univ-avignon.fr

We consider in this presentation the problem of computing a maximum flow on a temporal, or dynamic, graph. Our hypotheses are the following :

1. A temporal Graph $\mathcal{G}$ is composed of a sequence of t-graphs (or snapshot graphs) $G_1, \ldots, G_\theta, \ldots G_T$, with $G_\theta = (V, E_\theta)$
2. each time-arc $e = uv_\theta \in E_\theta$ has a capacity $c_\theta(uv)$
3. There is no travel time associated to the arcs.
4. Infinite storage is allowed on the nodes.

The goal is simply to send the maximum amount of flow from source s to sink t during these T time steps. Note that due to the hypotheses, a flow unit may cross arbitrary many successive arcs of G_θ, if the capacity constraints are respected on these arcs.

In the static case, the most efficient algorithms are based on Successive Shortest Paths or on Pre-flow principles. One may directly use these algorithms on the so-called expanded graph, built as follows :

— A super-source and a super-sink are added
— each node i is duplicated T times : $i_1, \ldots, i_T$
— each arc (i, j) of E_θ becomes an arc (i_θ, j_θ), with same capacity
— storage arcs are added between node i_θ and node $i_{\theta+1}$, with infinite capacity.

It is easy to verify that any flow of the expanded graph is a flow of the temporal graph, and reciprocally. However, the size of the expanded graph increases rapidly with T, and so does the complexity of the algorithms. Consider the fastest algorithm, Pre-flow variant using highest label priority. Its complexity on the expanded graph is $O(n^2\sqrt{m}T^2\sqrt{T})$ in time and $O(mT)$ in space.

Hence one important issue is : can we do better ?
In this presentation we shall provide some insights on this question, that remains open. Hopes are motivated by these two observations : The expanded graph is not any directed graph but exhibits special features; compact representation with capacity vectors should allow for new algorithms.

However, many tracks proved so far to be dead ends. We shall discuss on these negative results and look for new directions.