Bounding the Obstructions for Apices of Minor-closed Graph Classes

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A graph class $\mathcal{G}$ is *minor-closed* if every minor of a graph in $\mathcal{G}$ is also a member of $\mathcal{G}$. Given a graph class $\mathcal{G}$, its *minor obstruction set*, denoted by $\mathbf{obs}(\mathcal{G})$, is defined as the set of all minor-minimal graphs not in $\mathcal{G}$. According to the celebrated Robertson and Seymour's theorem [1], the obstruction set of every minor-closed graph class is finite.

The study of $\mathbf{obs}(\mathcal{G})$ for distinct instantiations of minor-closed graph classes $\mathcal{G}$ is an active topic in graph theory. There are many results in the literature that achieve a complete or partial identification of the obstruction set of different minor-closed graph classes.

Let $\mathcal{G}$ be a minor-closed graph class and k be a non-negative integer. We say that a graph G is a *k-apex* of $\mathcal{G}$ if G contains a et S of at most k vertices such that $G \setminus S$ belongs to $\mathcal{G}$. We denote by $\mathcal{A}_k(\mathcal{G})$ the set of all graphs that are k-apices of $\mathcal{G}$ and note that if $\mathcal{G}$ is minor-closed, then also $\mathcal{A}_k(\mathcal{G})$ is minor-closed. We prove that every graph in the obstruction set of $\mathcal{A}_k(\mathcal{G})$ has size at most $2^{2^{2^{2^{\mathsf{poly}(k)}}}}$, where poly is a polynomial function whose degree depends on the maximum size of a graph in $\mathbf{obs}(\mathcal{G})$. This bound drops to $2^{2^{\mathsf{poly}(k)}}$ when $\mathcal{G}$ excludes some apex graph as a minor, i.e., a graph that is an 1-apex to the class of planar graphs.

Références

[1] Neil Robertson and Paul D. Seymour. Graph Minors. XX. Wagner's conjecture. *Journal of Combinatorial Theory, Series B*, 92(2) :325–357, 2004. doi:10.1016/j.jctb.2004.08.001.