Vizing’s and Shannon’s Theorems for edge-defective colouring

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For a given multigraph $G$ and integer $d$, $\chi'_d(G)$ is the minimum size of a partition of $G$’s edges into subgraphs of maximum degree at most $d$. Note that for $d = 1$ this corresponds to the chromatic index of $G$.

We prove that for every integer $d$, every multigraph $G$ with maximum degree $\Delta$ satisfies $\chi'_d(G) \leq \lceil \frac{\Delta}{d} \rceil$ if $d$ is even and $\chi'_d(G) \leq \lceil \frac{3\Delta - 1}{3d - 1} \rceil$ if $d$ is odd and that these bounds are tight. This generalizes a result from Shannon in [1] stating that for any multigraph $G$, $\chi'_1(G) \leq \frac{3\Delta(G)}{2}$.

We also prove that for every simple graph $G$, $\chi'_d(G) \in \{\lceil \frac{\Delta}{d} \rceil, \lceil \frac{\Delta + 1}{d} \rceil \}$ and characterize the values of $d$ and $\Delta$ for which it is NP-complete to compute $\chi'_d(G)$, thus generalizing the infamous Vizing’s theorem (see [2]) and the corresponding NP-completeness result of Leven and Galil in [3].

Références