Vizing's and Shannon's Theorems for edge-defective colouring

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For a given multigraph G and integer d, $\chi'_d(G)$ is the minimum size of a partition of G's edges into subgraphs of maximum degree at most d. Note that for $d = 1$ this corresponds to the chromatic index of G.

We prove that for every integer d, every multigraph G with maximum degree Δ satisfies $\chi'_d(G) \leq \lceil \frac{\Delta}{d} \rceil$ if d is even and $\chi'_d(G) \leq \lceil \frac{3\Delta-1}{3d-1} \rceil$ if d is odd and that these bounds are tight. This generalizes a result from Shannon in [1] stating that for any multigraph G, $\chi'_1(G) \leq \frac{3\Delta(G)}{2}$.

We also prove that for every simple graph G, $\chi'_d(G) \in \{\lceil \frac{\Delta}{d} \rceil, \lceil \frac{\Delta+1}{d} \rceil\}$ and characterize the values of d and Δ for which it is NP-complete to compute $\chi'_d(G)$, thus generalizing the infamous Vizing's theorem (see [2]) and the corresponding NP-completeness result of Leven and Galil in [3].

Références

[1] C. Shannon, A Theorem on Coloring the Lines of a Network. *Studies in Applied Mathematics, Volume 28, Issue 1-4, April 1949, Pages 148-152.*

[2] V. G. Vizing, On an estimate of the chromatic class of a p-graph. *Diskret. Analiz.*, **3** :25-30, 1964). [In Russian]

[3] D. Leven and Galil Z. NP-completeness of finding the chromatic index of regular graphs. *Journal of Algorithms*, **4** :35–44, 1983.