# Improved pyrotechnics : Closer to the burning graph conjecture

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How fast can a rumor propagate in a graph ? One measure of that, introduced by Bonato, Janssen and Roshanbin [2], is the *burning number* $b(G)$ of a graph $G$. At step 1, we set a vertex on fire. At every step $i \geq 2$, all neighbours of a vertex on fire catch fire themselves, and we set a new vertex on fire. If at the end of step $k$ the whole graph is on fire, then the graph is $k$-burnable. The burning number of $G$ is defined to be the least $k$ such that $G$ is $k$-burnable. A graph with $n$ isolated vertices is trivially not $(n-1)$-burnable. We therefore focus on connected graphs. Paths are an interesting special case. For a path $P_n$ on $n$ vertices, it is not hard to check that $b(P_n) = \lceil \sqrt{n} \rceil$. When introducing the notion, Bonato et al. [2] conjectured that paths are, essentially, the worst case for the burning number of a graph.

**Conjecture 1** (Bonato et al. [2])**.** *Every connected graph $G$ satisfies $b(G) \leq \lceil \sqrt{|V(G)|} \rceil$.*

Conjecture 1 is only known to hold with a constant factor, we improved the best know bound by Land and Lu stating that $b(G) \leq \left\lceil \sqrt{3|V(G)|/2} \right\rceil + O(1)$.

**Theorem 2.** *Every connected graph $G$ satisfies $b(G) \leq \left\lceil \sqrt{\frac{4|V(G)|}{3}} \right\rceil + O(1)$.*

As a Corollary, we get that every graph on minimum degree at least 3 satisfies the conjecture up to an additive constant.

# Références

[1] Anthony Bonato. A survey of graph burning. *arXiv preprint arXiv :2009.10642*, 2020.

[2] Anthony Bonato, Jeannette Janssen, and Elham Roshanbin. Burning a graph is hard, 2015.

[3] Land, Max R and Lu, Linyuan. An upper bound on the burning number of graphs, *International Workshop on Algorithms and Models for the Web-Graph*, Springer, 2020.