J. Chalopin, V. Chepoi, U. Giocanti : Graphs with convex balls and groups acting on them

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Metric graph theory consists in studying graphs as metric spaces, when they are equipped with the shortest-path metric. During the last few decades, graph theoretic analogues of well-known geometrical properties have been intensively studied. In particular, it was shown that *systolic graphs* behave similarly to spaces with nonpositive curvature (also called CAT(0) spaces). Systolic graphs are graphs in which triangles are the only isometric cycles (a subgraph H of a graph G is isometric if the distances between vertices of H coincide with their distances in G). They generalize in a natural way chordal graphs, and were first studied separately in the eighties by Chepoi and Soltan [1] and by Farber and Jamison [2].

An important property of CAT(0) spaces and systolic graphs is that all balls are convex (a ball in a graph is the set of vertices at distance at most d from some vertex). A natural problem is to characterize the graphs that satisfy this property. These graphs are called *CB-graphs*, and were also introduced in the 80s (see [1], [2]).

We give a new characterization of CB-graphs as those graphs satisfying two classical metric properties, called the Interval Neighborhood Condition and the Triangle Pentagon Condition. We also give a local to global characterization of CB-graphs, giving a topological insight to their structure. Finally, we give a local characterization of some special clique paths in CB-graphs, implying thanks to a result of Swiatkowski that groups acting geometrically on CB-graphs are biautomatic (we will give all the necessary background on group actions during the talk).

Références

[1] V. Soltan and V. Chepoi, *Conditions for invariance of sets diameters under d-convexification in a graph*, English transl., Cybernetics **19(6)** (1983), 750–756.

[2] M. Farber and R. Jamison, *On local convexity in graphs*, Discrete Mathematics **66(3)** (1987), 231–247.