Hitting and Packing Rectangles with a Bounded Aspect Ratio

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Given a natural number r, a family of rectangles $\mathcal{R}$ is said to have *bounded aspect ratio* r if for any rectangle in $\mathcal{R}$ the ratio of the lengths of two perpendicular sides is at most r. Ahlswede and Karapetyan [1] stated, without providing proof, that for a family of axis-parallel rectangles with bounded aspect ratio r $\tau \leq 2(r+1)\nu$, where τ is the minimum number of points needed to hit all the rectangles in the family and ν is the maximum number of pairwise disjoint rectangles in the family.

We give an elementary proof of this result and generalize it to *not necessarily axis-parallel* rectangles, with a slightly weaker constant $\tau \leq 4(r+1)\nu$. Moreover, we observe that, in this more general setting, if one drops the hypothesis that the rectangles have a bounded aspect ratio, then $\frac{\tau}{\nu}$ can be arbitrarily large. Finally, we give more precise upper and lower bounds on τ for the particular case $r = 1$ that corresponds to families of squares.

Références

[1] R. Ahlswede and I. Karapetyan, *Intersection Graphs of Rectangles and Segments* , In : Ahlswede R. et al. (eds) General Theory of Information Transfer and Combinatorics. Lecture Notes in Computer Science, vol 4123. Springer, Berlin, Heidelberg, 1064–1065