Circular $(4-\epsilon)$-coloring of some classes of signed graphs

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A circular r-coloring of a signed graph (G, σ) is an assignment ϕ of points of a circle C_r of circumference r to the vertices of (G, σ) such that for each positive edge uv of (G, σ) the distance of $\phi(v)$ and $\phi(v)$ is at least 1 and for each negative edge uv the distance of $\phi(u)$ from the antipodal of $\phi(v)$ is at least 1. The circular chromatic number of (G, σ), denoted $\chi_c(G, \sigma)$, is the infimum of r such that (G, σ) admits a circular r-coloring.

This notion is recently defined by Naserasr, Wang, and Zhu. Among other results, they proved that for any signed d-degenerate simple graph $\hat{G}$, $\chi_c(\hat{G}) \leq 2d$. For $d \geq 3$, examples of signed d-degenerate simple graphs of circular chromatic number $2d$ are provided. But for $d = 2$ only examples of signed 2-degenerate simple graphs of circular chromatic number close enough to 4 are given, noting that these examples are also signed bipartite planar graphs. Using the notion of circular coloring and via a graph operation, we observe that the celebrated 4-Color Theorem could be restated as follows : If (G, σ) is a signed bipartite planar simple graph where vertices of one part are all of degree 2, then $\chi_c(G, \sigma) \leq \frac{16}{5}$.

Motivated from above, the classes of signed 2-degenerate simple graphs and signed bipartite planar simple graphs are of special interest in this study. We provide an improved upper bound of $4 - \frac{2}{\lfloor \frac{n+1}{2} \rfloor}$ for the circular chromatic number of a signed 2-degenerate simple graph on n vertices and an improved upper bound of $4 - \frac{4}{\lfloor \frac{n+2}{2} \rfloor}$ for the circular chromatic number of a signed bipartite planar simple graph on n vertices. We then show that each of the bounds is tight for any value of $n \geq 2$.